

ECE 257B: Principles of Wireless Networks

Class Exam Solution

1. The signal transmitted by a Base Station has a power of 1 mW at a distance of 1 m from the transmitter. Measurements have shown that the propagation exponent for the path loss is $n = 3$ up to a distance of 1 km and $n = 4$ beyond that. For adequate signal quality, a signal to noise ratio of at least 15 dB is required. Find the maximum cell radius if an AMPS receiver with bandwidth 30 kHz and total noise figure (including the antenna) $F = 6$ dB is used and (a) there is no shadowing, and (b) quality objective is to be met with probability 99% in the presence of log-normal shadowing with $\sigma = 6$ dB.

Solution: The signal power is 0 dBm at 1m and -90 dBm at $d_1 = 1$ km. The AMPS receiver has 30 kHz bandwidth, which corresponds to a thermal noise power of -129.2 dBm, which becomes -123.2 dBm when the 6 dB noise figure is added. Therefore, we have for $d > d_1$

$$SNR(d) = -40 \log \frac{d}{d_1} - 90 + 123.2 \text{ dB} = -40 \log \frac{d}{d_1} + 33.2 \text{ dB} \quad (1)$$

(a) Solving for $SNR(d) = 15$ dB yields $d = d_1 10^{0.455} \simeq 2.85$ km.

(b) With probability 99%, the shadowing attenuation does not exceed $2.3\sigma = 13.8$ dB. Solving for $SNR(d) = 15 + 13.8$ dB yields $d = d_1 10^{0.11} \simeq 1.29$ km.

2. You want to download a 5 Mbit file over a 10-Mbps Rayleigh fading channel in the 1800 MHz frequency band. You are traveling at 6 km/h and use 1000-bit packets. The fading margin of the channel is 20 dB. If any packet is received in error during the transfer, the operation is aborted. (a) What is the probability that you successfully transfer the file? Solve this by using both the Markov packet error model (use the table on the back of this sheet for r) and the level crossing rates (assume an exponential distribution for fading and non-fading durations). (b) If after a failure you wait a long time and then retry, what is the average number of attempts you will need to finally succeed? (c) Do you think doubling the packet size would help?

Solution: Wavelength at 1800 MHz is $1/6$ m, and 6 km/h = $10/6$ m/s, so that $f_D = v/\lambda = 10$ Hz. Duration of 1000-bit packets is $T = 0.1$ ms at the specified data rate, so that $f_D T = 0.001$. This value of $f_D T$ and the fading margin $F = 20$ dB correspond to $\varepsilon = P[\text{failure}] = 0.01$ and $r = P[\text{success}|\text{failure}] = 0.025$. The value of $q = P[\text{failure}|\text{success}]$ can be found from $\varepsilon = q/(q+r)$ to be $1/3960$, and $p = 1 - q$.

(a) The probability that the file is successfully sent is the probability that K consecutive slots are successful, where $K = 5000$ is the total number of 1000-bit packets in the 5-Mbit file. Therefore,

$$P[\text{file successfully sent}] = (1 - \varepsilon)p^{K-1} \quad (2)$$

since the channel state in the slot in which the first packet is sent has steady-state probability distribution. This value turns out to be about 0.28.

If level crossing rate analysis is to be used, note that the average crossing rate of level ρ is $N_R = \sqrt{2\pi} f_D \rho e^{-\rho^2}$, i.e., segments made up of one bad channel period and one good channel period have average length $1/N_R$. Since $1 - \varepsilon$ is the probability that the channel is in good state, the average duration of a good channel period is given by $(1 - \varepsilon)/N_R = (\sqrt{2\pi} f_D \rho)^{-1}$, which in the present case ($\rho = 0.1$, $f_D = 10$ Hz) amounts to about 0.4 s. Assuming that the duration of a good channel period is exponentially distributed, and given that the time needed to transmit the file is 0.5 s, the probability that the channel remains good for at least 0.5 s given that it is good at the beginning of the transmission is $e^{-0.5/0.4} \simeq 0.287$. The probability of successfully transmit the file is then given by this number multiplied by the steady-state probability that the channel is good, $1 - \varepsilon = 0.99$, and is equal to 0.283, which is consistent with the result we found previously, as expected.

(b) If after a failure we wait for a long time (with respect to the channel's coherence time), successive attempts are approximately independent of each other and the number of attempts needed to succeed is a geometric random variable whose mean is the inverse of the probability of success of a single attempt, i.e., about 3.57.

(c) By repeating the discrete Markov computation doubling T and dividing K by two gives a numerical result which is very close to the one found in (a). In the level crossing approach, nothing would change. Therefore, doubling the packet size would not help.

3. You have a bandwidth of 10 MHz (per direction) in a single-cell system and want to deploy a wireless telephone system. You have the following options: analog 30-kHz channels (as in AMPS), digital 30-kHz channels (each supporting 3 users as in IS-54), digital 200-kHz channels (each supporting 8

users as in GSM), or CDMA with 10-kbps user signals requiring an E_b/N_0 of 6 dB. (a) Compute the achievable capacity (in number of simultaneously active users per cell) for each case. (b) How do the results in (a) change if you use 120° sectorization? (c) How do the results in (b) change if instead of a single cell you have many? (Ignore voice activity factor in all of the above.) (d) For the above case of AMPS-like channelization with 7-cell reuse and 3 sectors per cell, suppose that you need to serve 900 users per cell. If the average duration of a call is 2 minutes, what is the maximum calling rate which guarantees a GOS of 2%?

Solution:

(a) The number of full-duplex channels available is as follows: AMPS: $10000/30 = 333$; IS-54: $10000/30 \times 3 = 1000$; GSM: $10000/200 \times 8 = 400$. For CDMA, E_b/N_0 can be found as

$$\frac{E_b}{N_0} = \frac{W/R}{K - 1} \quad (3)$$

where $W = 10$ MHz, $R = 10$ kbps, and K is the number of users. In order to guarantee that $E_b/N_0 \geq 6$ dB = 4, a maximum of about 250 users can be simultaneously supported.

(b) In the first three cases, nothing changes, since sectorization does not increase the number of channels per cell. In CDMA, a threefold gain in capacity is achieved, and 750 users can now be simultaneously active.

(c) In the first three cases, some frequency reuse plan must be used, so that the number of channels per cell is correspondingly reduced (e.g., by a factor of 7). In CDMA, no frequency reuse plan is needed, and the effect of the multicell structure is to increase the interference by about 50%. Therefore, the number of simultaneously active users which can be supported in the CDMA system is now 500.

(d) Each sector has now $333/3 = 111$ channels, i.e., some sectors will have 111 and some others will have 112. For 111 channels, GOS of 2% is achieved at 10 Erl (9 Erl for 111 channels). If we have 900 users per cell, there will be 300 users per sector sharing the 111 channels. In order for the GOS to be met, each user can generate up to $1/30$ Erl which, with 2 minutes as the average call duration, corresponds to a calling rate of one call per hour.