ECE 257B: Principles of Wireless Networks
Solution of Homework assignment # 2

1. The approximated SIR for cluster size N and S sectors is computed as $S(3N)^{n/2}/6$. Therefore, we have to find the minimum value of N such that $S(3N)^{n/2}/6 \geq 15$ dB = 31.6. We find the following for $n = 4$:

number of sectors	minimum N	SIR
1	7	18.7 dB
3	3	16.1 dB
6	3	19.1 dB

   Note that the case $N = 3, S = 3$ has little margin over the required 15 dB, so that a more accurate computation may result in a different (larger) N.

   The relative merit of the three scheme depends on the trunking efficiency achievable, which in turn depends on the number of channels and GOS (unspecified). In order to accurately study the trade-off, one should plot the amount of traffic which can be carried vs. the total number of channels assigned to the system (for a given GOS). See figure 1 for GOS = 0.01. It is clear that for reasonable values of the system parameters the choice $S = 3$ gives the best results. Obviously, the case $S = 6$ will never perform better than $S = 3$ since they result in the same value of N (as the number of channels goes to infinity, they will perform the same).

2. Same as before with $n = 3$.

number of sectors	minimum N	SIR
1	12	15.6 dB
3	7	16.8 dB
6	4	16.1 dB

   As before, when the values of the SIR are close to the threshold, a more accurate computation may result in a higher value of N. Figure 2 shows the relative performance for GOS = 0.01. The curve crossovers occur for a small number of channels (less than 80 in all cases), so that for typical situations the case $S = 6$ will in fact maximize the performance, as the increased number of channels per cell overweighs the loss in trunking efficiency due to sectorization.

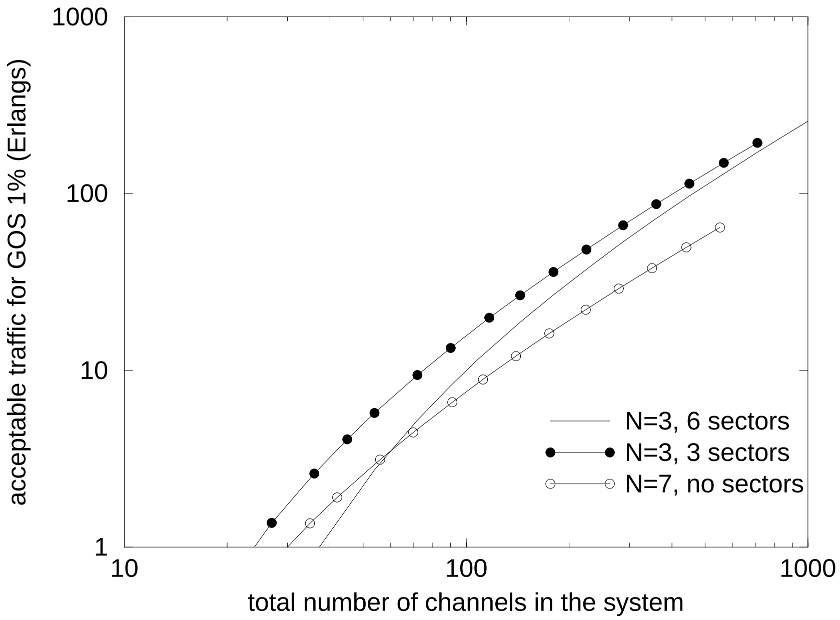

Figure 1:

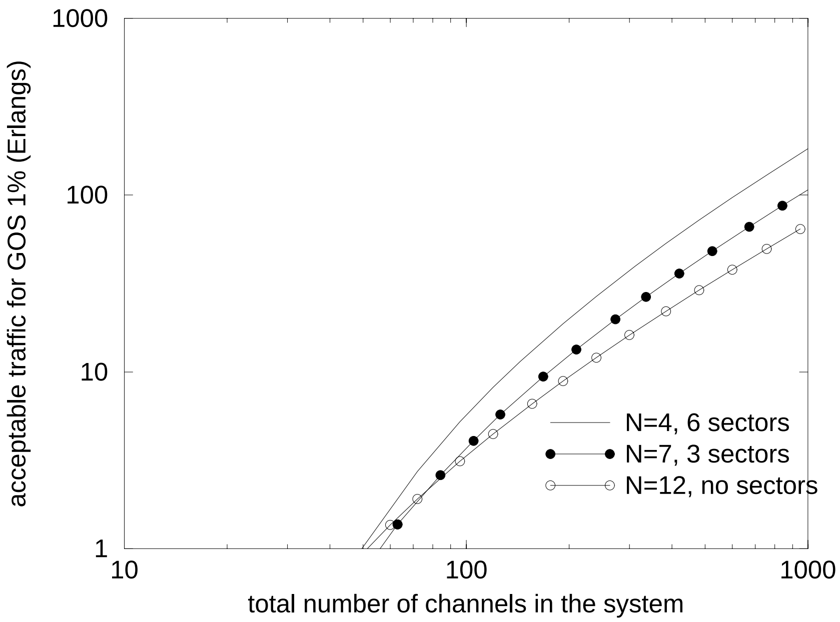

Figure 2:

3. In order to have -100 dBm of interfering power at distance d with 0 dBm at distance $d_0 = 1$ m and $n = 3$, we must have $30 \log(d/d_0) = 100$, i.e., $d = 10^{10/3} d_0 \simeq 2150$ m. This is the reuse distance, which is related to the cell radius by $R_u = \sqrt{3N}R$, so that $R = R_u/\sqrt{21} \simeq 470$ m for $N = 7$ and $R = R_u/\sqrt{12} \simeq 620$ m for $N = 4$.

4. With the above values, we have a cell area of $3\sqrt{3}R^2/2 \simeq 0.574$ km^2, which corresponds to $0.574 \times 9000 \times 1/60 \simeq 86$ Erl/cell (each user generates 1/60 Erl). With 90 channels, we have a probability of being queued equal to 0.58 (from Erlang-C formula), and a probability that a non-zero queueing delay exceeds 20 s equal to $\exp(-(90-86)20/60) \simeq 0.27$. The probability of a call being queued for more than 20 s is then the product of the two, and is equal to 0.16.

5. (a) One 5-minute call every 20 minutes = 0.25 Erl.
(b) Since the Erlang-B formula gives a traffic smaller than 0.25 for one channel and GOS = 0.01, only one user can be supported (note that at least one can always be supported)
(c) GOS = 0.01 and 5 channels: 1.36 Erl (from Table 2.4), i.e., up to five users (six would be 1.5 Erl).
(d) 10 users = 2.5 Erl, with 5 channels we have GOS = 7%, which of course exceeds our objective but is not exceedingly high (typical values range between 1% and 5%).

6. (a) Full duplex, with 30 kHz per direction.
(b) The subscriber unit transmits 45 MHz below, i.e., at 835.560 MHz.
(c) 21 control and 395 voice channels per carrier.
(d) Dividing channels equally among sectors, we will have 17 sectors with 19 channels and 4 sectors with 18.
(e) 7-cell: $\sqrt{21} = 4.6$; 4-cell: $\sqrt{12} = 3.46$.

7. The SIR decrease when going from $N = 9$ to $N = 7$ is a factor of $(9/7)^{n/2} = 1.87 \simeq 2.7$ dB for $n = 5$, and therefore it is within the 4-dB margin provided by the new receiver.

8. In the pedestrian case we have $f_D = v/\lambda = 2.5$ Hz and $T = 1$ ms, so that $f_D T = 0.0025$. In the vehicular case we have $f_D = 50$ Hz and $T = 5$ ms, so that $f_D T = 0.25$. From the table handed out in class, we have the following:

	5 dB	10 dB	15 dB	20 dB	25 dB
r_{ped}	0.009519	0.018931	0.034842	0.062601	0.111515
r_{veh}	0.678185	0.881383	0.960499	0.987298	0.995962

The length of a run of erroneous packets is a random variable with geometric distribution and mean value $1/r$, so that the probability that it is larger than τ is $(1-r)^\tau$. Therefore, the minimum value of the time-out (in packets) which guarantees that no more than a fraction ζ of Rayleigh fading occurrences cause connection resetting is such that $(1-r)^\tau \leq \zeta < (1-r)^{\tau-1}$, i.e., $\tau = \lceil \log \zeta / \log(1-r) \rceil$, and is given in the table below:

	5 dB	10 dB	15 dB	20 dB	25 dB
pedestrian, $\zeta = 0.01$	482	241	130	72	39
pedestrian, $\zeta = 0.05$	314	157	85	47	26
vehicular, $\zeta = 0.01$	5	3	2	2	1
vehicular, $\zeta = 0.05$	3	2	1	1	1

Note: The correct interpretation of the problem was to guarantee that errors due to Rayleigh fading cause connection resetting 1% or 5% of the time, i.e., you had to consider the *conditional* probability distribution of the number of consecutive erroneous packets *given that an error occurs* (that's why the steady-state packet error probability is not used here).

A different way of doing this is as follows. The average length of a fade of a certain depth is given by Equation (4.84) (please remember that ρ is a voltage, not a power and therefore 10 dB means $\rho = \sqrt{0.1} = .316$). If we approximate the distribution of the fade duration as exponential with that mean value, we obtain $\tau = \lceil \overline{\tau} \log \zeta \rceil$. By using this approach, we obtain the following:

	5 dB	10 dB	15 dB	20 dB	25 dB
pedestrian, $\zeta = 0.01$	487	245	133	74	42
pedestrian, $\zeta = 0.05$	317	159	87	49	27
vehicular, $\zeta = 0.01$	25	13	7	4	3
vehicular, $\zeta = 0.05$	16	8	5	3	2

Note that assuming an exponential distribution for the fades is somewhat arbitrary, but it is basically equivalent to assuming a Markov model if the fade duration is significantly longer than a packet. If this condition is not met, then the exponential approximation is not good. This is clearly seen by comparing the two tables.