ECE 257B: Principles of Wireless Networks
Solution of Homework assignment # 3

1. (a) Raw rate per user is $270.833/8 \simeq 33.85$ kbps.

   (b) The efficiency is $(33.85 - 10.1)/33.85 \simeq 70\%$.

2. Raw data rate is $48.6/3 = 16.2$ kbps.

3. (a) In each slot, there are $6 + 6 + 28 = 40$ bits of overhead. The framing efficiency is then $(324 - 40)/324 \simeq 88\%$. If control traffic is considered as overhead (i.e., not useful information from the phone call's point of view), the efficiency is reduced to about 80%.

   (b) With half-rate speech coding, each user gets one (instead of two) slots per frame, i.e., the data rate is 8.1 kbps. Framing efficiency is the same as in (a).

4. We have for a CDMA system
$$\frac{E_b}{N_0} = \frac{W/R}{N-1}. \tag{1}$$
   Since here we have $R = 13$ kbps, $N = 100$, and we require an E_b/N_0 of 20 dB = 100, we obtain $W = 130$ MHz or a chip rate of 130 Mbps (assuming a modulation efficiency of 1b/s/Hz for simplicity), i.e., a spreading factor of 10,000.

5. Voice activity reduces the amount of interference accordingly, i.e., we need a spreading factor which is 40% of the value found in the previous exercise, and $W = 52$ MHz.

6. Sectorization further reduces the interference (and therefore the required bandwidth) by a factor of 3, so that $W = 17.3$ MHz.

7. (a) The packet duration is 0.1 ms in this case, so that the normalized offered traffic (average number of packet arrivals in an interval equal to the packet transmission time) is $R = 0.1$. This value of R corresponds to a throughput of $Re^{-2R} \simeq 0.082$.

   (b) Maximum throughput occurs for $R = 0.5$, i.e., five times as many packet arrivals during packet transmission time. Optimum packet size is then five times as it was in (a), i.e., 5000 bits.

8. The denominators in Eq. (21) should have $j\mu_H$ instead of just μ_H.

9. For Poisson arrivals, the distribution of the number of transmissions in a given slot (equal to the number of arrivals in the previous slot) is found as

$$p_n = P[\text{exactly } n \text{ arrivals in a slot}] = \frac{\lambda^n e^{-\lambda}}{n!} \quad (2)$$

where λ is the normalized arrival rate (R in Rappaport's book). Since the capture probability C_n is the average throughput conditioned on the number of simultaneous transmissions in a slot, n, the unconditional average throughput is found from the total probability theorem as

$$S = \sum_{n=0}^{\infty} p_n C_n. \quad (3)$$

In our case, we have $C_n = n(1+b)^{-(n-1)}$, and we obtain

$$S = \sum_{n=1}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} n(1+b)^{-(n-1)} = \lambda e^{-\lambda} \sum_{j=0}^{\infty} \frac{(\lambda/(1+b))^j}{j!} = \lambda e^{-\lambda} e^{\lambda/(1+b)} = \lambda e^{-\lambda/(1+1/b)}. \quad (4)$$

The maximum of this function is equal to $S^* = (1+1/b)e^{-1}$, and occurs for $\lambda^* = 1+1/b$. We have $S^* = 0.460, \lambda^* = 1.25$ for $b = 6$ dB = 4, and $S^* = 0.405, \lambda^* = 1.1$ for $b = 10$ dB = 10.

For $C_1 = 1, C_n = Q^n, n > 1$, we obtain

$$S = \lambda e^{-\lambda} + \sum_{n=2}^{\infty} \frac{\lambda^n Q^n e^{-\lambda}}{n!} = \lambda e^{-\lambda} + e^{-\lambda}[e^{\lambda Q} - 1 - \lambda Q] = \lambda(1-Q)e^{-\lambda} + e^{-\lambda(1-Q)} - e^{-\lambda} \quad (5)$$

which is maximum for the unique λ^* which satisfies

$$\lambda^* = 1 + (1-Q)^{-1} - e^{\lambda^* Q} \quad (6)$$

(obtained from $\partial S/\partial\lambda = 0$).

10. (a) If $round(x)$ is the closest integer to x, we have

$$P_T(x) = \frac{P_0 R^4}{K}\left(\frac{x}{R} - round\left(\frac{x}{R}\right)\right)^4, \quad (7)$$

plotted in Figure 1.

(b) We have

$$P_R(x) = K x^{-4} P_T(x) = \begin{cases} P_0 & \text{if } |x| \le R/2 \\ P_0\left(1 - round\left(\frac{x}{R}\right)\left(\frac{R}{x}\right)\right)^4 & \text{otherwise} \end{cases}, \quad (8)$$

plotted in Figures 2 and 3.

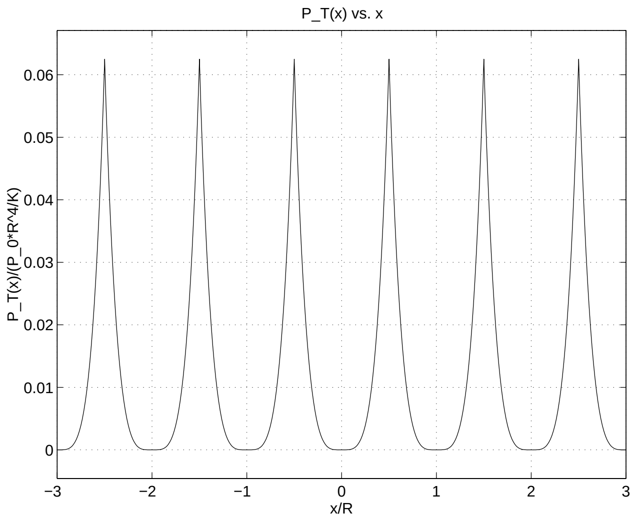

Figure 1:

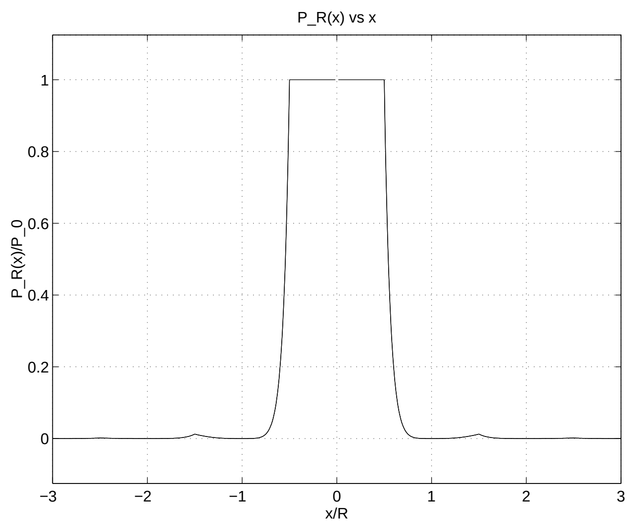

Figure 2:

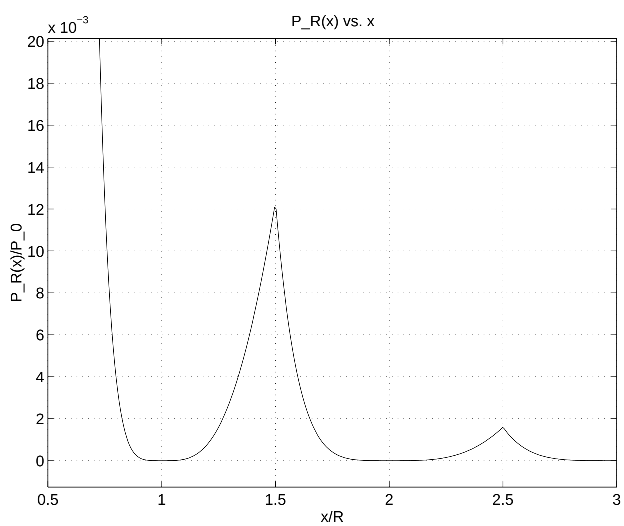

Figure 3:

(c) We know that the mobiles are equally spaced, but we do not know their relative position with respect to the base stations. If we assume that this "phase" is uniformly distributed, then the probability that a small segment contains a user is given by αdx, so that the average interference at $BS(0)$ contributed by that small segment is $P_R(x)\alpha dx$, and the total interference power is just

$$I = \alpha \int_{-\infty}^{\infty} P_R(x)dx = I_0 + 2\sum_{i=1}^{\infty} I_i \tag{9}$$

where $I_i = I_{-i}$ is the interference coming from cell i and is given by

$$I_i = \alpha \int_{(i-0.5)R}^{(i+0.5)R} P_R(x)dx = \alpha \int_{-R/2}^{R/2} P_0 t^4 (iR+t)^{-4} dt = \beta_i \alpha R P_0. \tag{10}$$

The coefficients

$$\beta_i = \frac{1}{R}\int_{-R/2}^{R/2} t^4 (iR+t)^{-4} dt \tag{11}$$

are independent of R and decay as i^{-4}, so that the infinite sum can be truncated for a small number of terms (e.g., three or four). Here are some values (for $i = 0$ the received power is a constant):

i	0	1	2	3	4
β_i	1	0.0623	0.00120	0.000187	0.0000545

Therefore, we have

$$I = \left(\beta_0 + 2\sum_{i=1}^{\infty} \beta_i\right) \alpha R P_0 \simeq 1.1276 \alpha R P_0. \tag{12}$$

The power received from the intended user is identically equal to P_0, regardless of the user's location, because of the use of power control. Therefore, the average Signal-to-Interference Ratio is

$$\frac{S}{I} = \frac{1}{1.1276\alpha R}. \tag{13}$$

(d) Since $W/R = 500$, we have

$$\frac{E_b}{N_0} = 500\frac{S}{I}, \tag{14}$$

which is ≥ 10 dB if $1.1276\alpha R \leq 50$, i.e., $\alpha R \leq 44.34$. An average of about 44 active users per cell can be admitted.

11. (a) Since no distinction is made between calls, all must be blocked with probability not exceeding 0.2%, i.e., by applying Erlang-B with traffic load 30 Erl the minimum number of channels which guarantees this GOS is $C = 46$ (which yields GOS = 0.15%; with 45 channels you would get 0.23%).

    (b) From the theory given in Hong and Rappaport (Section IV.A) for the prioritized handoff analysis, you have to find the minimum value of C such that there exists a value of C_h simultaneously giving $P_B \leq 0.02, P_{fh} \leq 0.002$. The answer is $C = 42, C_h = 2$, for which we have $P_B = 0.0187, P_{fh} \leq 0.00083$, whereas for $C = 41$ no value of C_h meets both performance criteria (in this case, the values of (P_B, P_{fh}) are (0.01,0.01) for $C_h = 0$, (0.018,0.0035) for $C_h = 1$, and (0.026,0.0012) for $C_h = 2$).